

Short Communication

Analytical Solution for Adiabatic Surface Temperature (AST)

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Abstract. In this contribution an analytical solution of the equation of an ideal surface of a perfect insulator is introduced. The new solution bases on the solution of the heat balance equation, which is fourth degree polynomial. The resulting expression for adiabatic surface temperature (AST) is discussed and verified. This solution can be easily incorporated, with negligible computational cost, inside the computational fluid dynamics solvers for fire simulations. Finally, since AST is close to the temperature measured by plate thermometers, AST is easy to measure in any arbitrary point in a computational domain. This makes it possible to validate numerical results with the temperatures measured by plate thermometers in experiments. Furthermore, an expression, which is given in the end of the article, describes the physical quantity with a closed-form solution. This may be potentially used for analyses performed for better understanding of physical process, e.g. during the experiments involving localized fires.

Keywords: Adiabatic surface temperature, Fire safety engineering, Heat transfer, Plate thermometer, Gas–solid interface

1. Introduction

Since Professor Ulf Wickström introduced the concept of adiabatic surface temperature (AST) to the fire science community in 2007 [1], it appears to be a very efficient way for expressing heat exposure of the solid surfaces both in real experiments, using plate thermometers [2–4] which measure the temperature that is close to the AST [5], and in numerical analyses [6–9]. Adiabatic surface temperature can be also used as a single thermal boundary conditions when calculating temperature of structures exposed to fire [10, 11], which is one of the biggest advantages of this concept. Thus, such computational fluid dynamics (CFD) numerical codes as fire dynamics simulator incorporated AST as the variable that can be obtained at solid surfaces [12]. Nonetheless, simulating fires is computationally very expensive and the size of numerical grid usually does not allow to accurately model structural elements. In case of steel sections they usually even do not exist in CFD models, and then it is not possible to explicitly describe the thermal conditions at

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their section surfaces. In this case, many researches extract necessary quantities from CFD analyses, such as: incident radiations, gas velocities, gas temperatures, which finally ends up with a big amount of data that have to be posteriorly processed [13]. This problem has been already discussed by Joakim Sandström in his thesis [14]. He developed a method for numerical evaluation of AST at arbitrary points and directions in the computational domain, not necessarily connected to any actual surface. That method requires additional computation time, in order to numerically calculate the AST by solving the heat balance equation. In this paper, an analytical solution for the same problem is introduced, scientifically discussed and verified. This analytical approach, resulting in closed-form solution for AST, is advantageous not only from the numerical point of view, but may be also beneficial for future scientists examining the influence of natural fires on the dynamics of heat distribution in space.

2. Heat Balance Equation

There are two contributions of the total (net) heat flux \dot{q}_{net}'' to a surface: convective heat flux \dot{q}_{conv}'' and radiative heat flux \dot{q}_{rad}'' . The sum $\dot{q}_{net}'' = \dot{q}_{rad}'' + \dot{q}_{conv}''$ can be obtained by the following formula:

$$\dot{q}_{net}'' = \varepsilon(\dot{q}_{inc}'' - \sigma T_s^4) + h_c(T_g - T_s) \quad (1)$$

where ε is the emissivity of the surface, \dot{q}_{inc}'' the incident radiation, σ the Stefan–Boltzmann constant, h_c the convective heat transfer coefficient. Finally T_g is the gas temperature and T_s is the surface temperature.

The total net heat flux to an ideal surface of a perfect insulator is by definition zero, thus introducing concept of AST T_{AST} , following expression has to be fulfilled (according to Wickström [1]):

$$\varepsilon(\dot{q}_{inc}'' - \sigma T_{AST}^4) + h_c(T_g - T_{AST}) = 0 \quad (2)$$

After elementary algebraical operations, Eq. (2) can be written as:

$$\varepsilon\sigma T_{AST}^4 + h_c T_{AST} + (-\varepsilon\dot{q}_{inc}'' - h_c T_g) = 0 \quad (3)$$

what is the fourth order polynomial equation with T_{AST} as variable, that may be written in the form:

$$aT_{AST}^4 + bT_{AST} + c = 0 \quad (4)$$

where subsequent coefficients of the polynomial are: $a = \varepsilon\sigma$, $b = h_c$, $c = -\varepsilon\dot{q}_{inc}'' - h_c T_g$. Since a is always positive for gray and black bodies ($\varepsilon > 0$), Eq. (4) has always four roots, but some of the roots may be complex numbers.

3. Solution

In order to solve an Eq. (4), the WolframAlpha on-line system for symbolic and numeric mathematics has been used [15]. Finally the output from the computational engine is simplified and the solutions of Eq. (4) can be written in the following forms:

$$T_{AST}^{(1st\ root)} = \frac{1}{2} \left(M - \sqrt{\frac{-2b}{aM} - M^2} \right) \quad (5)$$

$$T_{AST}^{(2nd\ root)} = \frac{1}{2} \left(M + \sqrt{\frac{-2b}{aM} - M^2} \right) \quad (6)$$

$$T_{AST}^{(3rd\ root)} = \frac{1}{2} \left(-M - \sqrt{\frac{2b}{aM} - M^2} \right) \quad (7)$$

$$T_{AST}^{(4th\ root)} = \frac{1}{2} \left(-M + \sqrt{\frac{2b}{aM} - M^2} \right) \quad (8)$$

where

$$M = \sqrt{\frac{\beta}{\alpha} + \frac{\alpha}{\gamma}} \quad (9)$$

Using coefficients a , b and c as previously introduced, α , β and γ are obtained using following formulas:

$$\alpha = \left(\sqrt{3} \sqrt{27a^2b^4 - 256a^3c^3} + 9ab^2 \right)^{\frac{1}{3}} \quad (10)$$

$$\beta = 4 \left(\frac{2}{3} \right)^{\frac{1}{3}} c \quad (11)$$

$$\gamma = (18)^{\frac{1}{3}} a \quad (12)$$

4. Discussion

From above consideration, it can be seen that there are four equivalent solutions resulting with four potential values for adiabatic surface temperature. Those four solutions, without the scientific deliberation, are only unprofitable mathematical formulas. In order to give the physical sense, it is necessary to come back to the physical meaning of the coefficients within those formulas. Since $a = \varepsilon\sigma$ and, from the definition, $(\varepsilon, \sigma) > 0$, the coefficient a is always positive. Similarly, since $b = h_c$, to have physical meaning, it must be positive (for $h_c = 0$ we have trivial solution explained in next section). On the other hand $c = -(\varepsilon\sigma T_r^4 + h_c T_g)$ is valid for temperatures in Kelvin and always gives negative value.

Considering now relations (10–12) it is clearly seen, that for positive values of a and b , and negative value of c , the coefficients α and γ are clearly positive, and coefficient β is negative. Let us now look at the coefficient M . If M takes the real value, then the expression under square root must be non-negative. At the same time, expressions (5–8) may be evaluated only if M is positive. Thus, after standard algebraic operations M can be rearranged to:

$$M = \sqrt{\frac{\beta}{\alpha} + \frac{\alpha}{\gamma}} = \sqrt{\frac{\beta\gamma + \alpha^2}{\alpha\gamma}} \quad (13)$$

Since $\alpha\gamma > 0$, the value of $M > 0 \iff \beta\gamma + \alpha^2 > 0$. Substituting expressions (10–12) into that right-hand side inequality we obtain:

$$\beta\gamma + \alpha^2 > 0 \iff 4\left(\frac{2}{3} \cdot 18\right)^{\frac{1}{3}} ac + \left(\sqrt{3}\sqrt{27a^2b^4 - 256a^3c^3} + 9ab^2\right)^{\frac{2}{3}} > 0 \quad (14)$$

Let us now conduct basic mathematical operations on the inequality (14) as following:

$$\begin{aligned} \left(\sqrt{3}\sqrt{27a^2b^4 - 256a^3c^3} + 9ab^2\right)^{\frac{2}{3}} &> -4(12)^{\frac{2}{3}} ac \\ \left(\sqrt{3}\sqrt{27a^2b^4 - 256a^3c^3} + 9ab^2\right)^2 &> -768a^3c^3 \\ 3(27a^2b^4 - 256a^3c^3) + 2\sqrt{3}\sqrt{27a^2b^4 - 256a^3c^3} \cdot 9ab^2 + 81a^2b^4 &> -768a^3c^3 \\ 81a^2b^4 - 768a^3c^3 + 2\sqrt{3}\sqrt{27a^2b^4 - 256a^3c^3} \cdot 9ab^2 + 81a^2b^4 &> -768a^3c^3 \\ 162a^2b^4 + 2\sqrt{3}\sqrt{27a^2b^4 - 256a^3c^3} \cdot 9ab^2 &> 0 \end{aligned} \quad (15)$$

Finally, for physical values of a , b and c , inequality (15) is always true, so that coefficient M is always positive. Coming back now to expressions (5–8) it is clearly visible that: expressions (5) and (6) have no real evaluation; expression (7) gives negative (unphysical) value; the only expression that may give real and physical result is expression (8).

Thus, let us now recall expression (8) as the exact solution for evaluation of adiabatic surface temperature:

$$T_{AST} = \frac{1}{2} \left(-M + \sqrt{\frac{2b}{aM} - M^2} \right) \quad (16)$$

Eq. (16) is finally proven by tests over a range of ε/h_c ratios. In Figure 1, it is shown, that the dependence between AST and parameters of Eq. (2) are the shape of those referred by Sandström [14]. Additionally, quantitative comparison between AST obtained with solution (16) and approximate solution obtained using Newton–Raphson method is considered in next section.

5. Exact Versus Approximate Solution

The biggest advantage of using a closed-form exact solution is no need for performing time consuming, iterative computations, in order to obtain the result. Currently, AST is calculated mostly using the Newton–Raphson method. According to author's tests, AST can be approximated within no more than three iterations, with the error less than 0.5 °C. This is valid for different configurations of gas and incident heat flux temperatures, when the incident heat flux temperature is taken as the first guess and the derivative of the function is calculated directly (not approximated). That means, using a closed-form solution can speed-up the process of computations about three times; the solution is not sensitive to “the first guess” and the approximation of derivative; finally solution is pure physical, not affected by approximation error.

Comparison between exact and approximate solution is shown in Figure 2 (graphs overlap).

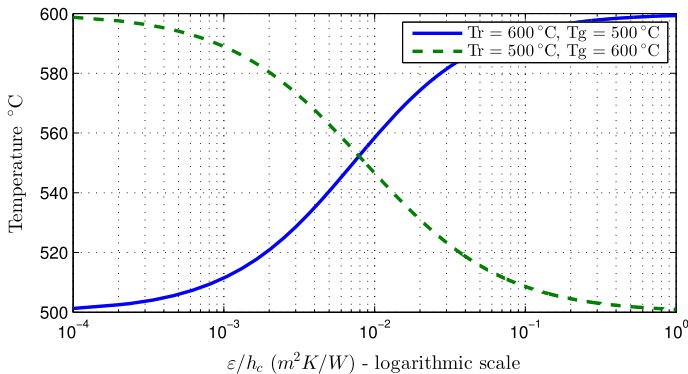


Fig. 1. Relationship between AST and ε/h_c .

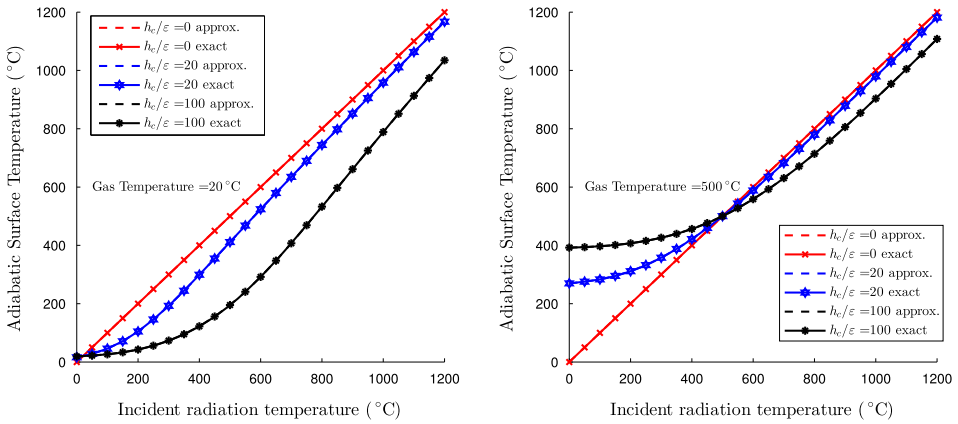


Fig. 2. Comparison between exact and approximate solutions for $T_g = 20^\circ\text{C}$ and $T_g = 500^\circ\text{C}$ (graphs overlap).

6. Fire Safety Engineering Application

The usefulness of AST concept in fire safety engineering (FSE) is related to the heat transfer calculation. During the experiments, the crucial issue is the determination of heat boundary conditions on the specimen surface. Wickström, in [16], summarizes how important the determination of heat flux is and how many problems are related with measurements of the physical quantities in experiments. Thus, the plate thermometers are used as the devices that can measure the exposure of a surface both to convection and radiation. A good and simple example of an application of AST concept is given by Byström et al. [17]. Authors compare a temperature data measured using plate thermometer and two types of thermocouples in a test carried out in cone calorimeter with burning specimen under the cone shape radiation panel. Byström summarizes, that plate thermometer is sensitive both to convection and radiation in a similar way as real specimen and the AST can be measured using plate thermometers even under harsh fire conditions. In the same way, one can imagine the test, where the AST is to be specified at a specimen surface with the conditions given by incident radiation and gas temperature. Then AST may be calculated directly from Eq. 16 and can be used as a single quantity for heat transfer calculations.

7. Final Remarks

An analytical solution of the polynomial expressing the heat balance between the total (net) heat flux approaching the ideal surface of perfect insulator is introduced. The resulting expressions for roots of heat balance equation are discussed and the only expression having physical meaning is chosen as the one, that describes adiabatic surface temperature. Results obtained by proposed solution are quantitatively and qualitatively checked with respect to previous researchers

findings. It is shown, that since the approximate iterative solution is sensitive to “the first guess” and the approximation of derivative, proposed analytical solution gives exact results for the whole range of applications. This solution can be easily incorporated, without any computational cost, inside the computational fluid dynamics solvers for fire simulations. Finally, enabling measurements of AST in any arbitrary points and directions of computational domain, makes it possible to validate numerical results with the results given by plate thermometers in real experiments with satisfactory accuracy. Because the given expression describes the physical quantity in an exact way, it may be potentially used for analyses performed for better understanding of physical process, e.g during the real experiments.

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